In one litre of water we replace each molecule with a litre water. From that we make a planet...

Reinko Venema.

10 July 2014

Introduction. It seems that weirdly enough a whole lot of teenagers are actually reading this website.
Well I consider that being a compliment because the last time I looked young was from before the invention of the stone age...

Therefore no hardcore math but some easy going calculations around the number of Avogadro.
The number of Avogadro is a brilliant invention because it relates the number of nuclear particles in an atom or molecule to grams.
The symbol for this number is N_A.
Example: Hydrogen molecules come in pairs H_2 and since every H atom contains one proton, the molecule contains two protons and as such if we have N_A molecules of H_2 that weighs 2 grams.

Simple isn't it?
Since protons and neutrons are very very small, the number of Avogadro is very large. It is

$$N_A = 6.02214 \cdot 10^{23}$$

And that is huge because 10^{23} is a 1 followed by 23 zero's.
Actually there is a tiny mass difference between a proton and a neutron so actually there are two numbers of Avogadro, but we simply neglect that tiny tiny difference.

Ok, how many protons and neutrons are in one molecule of water H_2O ?

That is very simple; oxygen ranks number 8 in the periodic system of elements so it has 8 protons.
Small atoms like oxygen always have the same number of protons and neutrons, with large atoms the situation is extremely more complicated.
But for oxygen we have 8 protons and 8 neutrons, the weight of the electrons is neglectable since it is about 2000 times as small as the nuclear particles.

Adding it up, one water molecule has 10 protons and 8 neutrons. The number of Avogadro is also called a mol (from molair), a mol of water molecules weighs thus 18 gram.
Let's denote the number of water molecules in one litre by X.
How many water molecules are there into one litre?
Very simple; one litre of water weighs 1000 gram.
So that is $1000/18 \approx 55.6$ mol.
That gives

$$X = 55.6 \cdot 6.02214 \cdot 10^{23} = 3.34831 \cdot 10^{25}$$

So think about that for a few seconds because it is such a huge huge number and just for drinking you need a few litres every day...

Ok, thus far we know the value of X, but what is the volume of X litres of water?
And since $3.34831 \cdot 10^{25}$ litres of water is an amazing amount of water, it is most handy to express this in cubic kilometres.

So we want to know how much cubic kilometres is filled by $3.34831 \cdot 10^{25}$ litres of water?
One kilometre is one thousand meters, so on cubic kilometre is one billion cubic meters.
And every cubic meter is one thousand litres.
Therefore

$$\text{one cubic kilometre} = 10^9 \text{ cubic meters } = 10^{12} \text{ litres}$$

And X litres of water would amount to

$$3.34831 \cdot 10^{25}/10^{12} = 3.34831 \cdot 10^{13} \text{ cubic kilometres}$$

This amount of cubic kilometres can be related to the radius of our water planet. For that we use the inverse relation between the volume V of a sphere and it's radius r

$$V = \frac{4}{3}\pi r^3$$

This is easy to invert

$$r^3 = \frac{3V}{4\pi} \text{ so that } r = \sqrt[3]{\frac{3V}{4\pi}}$$

The volume of water is of course $V = 3.34831 \cdot 10^{13}$ km^3
So the radius of our water planet becomes

$$r = \sqrt[3]{\frac{3 \cdot 3.34831 \cdot 10^{13}}{4\pi}} = \sqrt[3]{7.9935 \cdot 10^{12}} = 19994 \text{ km}$$

How large is that?
The radius of the earth at the equator is 6378 km (remark that our planet is spinning so it is not a perfect sphere). The radius of the water planet is just over 3 times as large:

$$\frac{19994}{6378} \approx 3.1348$$

and therefore the volume of the water planet is about

$$3.1348^3 \approx 31 \text{ times as big as the earth}$$

Summary What have we done?
We estimated the number of molecules in one litre of water.
We named that number X
We calculated the volume of X bottles of water.
That was about 31 times the volume of the earth...

I hope you now understand how small water molecules actually are.

End of the summary.